**Risky Portfolios**

* Return vs. Volatility (for Portfolios)
* Takeaway
	+ Portfolios have less risk than the stocks that make them up
	+ Risk has 2 categories:
		- Market Risk
			* Whole economy
			* Involves large, macroeconomic forces that essentially all firms are affected by
				+ Can’t get rid of a shared risk by holding more than one stock
			* Also known as systematic or Nondiversifiable risk
		- Firm-specific risk
			* Specific to the company
			* Generally, within the firm’s control
				+ Can offset with stocks that don’t bear the same risk
			* Also known as unsystematic or diversifiable risk
* Interpreting the Types
	+ Total risk = systematic + unsystematic risk
* Portfolio Weights
	+ Fraction of the total investment in the portfolio held in each individual investment is known as the portfolio weight
	+ Weight must add up to 1 (100%)
		- Wi= Value of investment i / total value of portfolio
	+ Weights are not probabilities: don’t have to be positive and each individual weight can be >1
		- When you short, you have a negative weight in that stock
		- When you buy on margin you can have over 100% since you borrowed money
* Example: Weights
	+ You have a portfolio worth $200,000. Of that, $40,000 is invested in XYZ Corp. What is your weight in this stock?
	+ What is the weight of the other stocks in your portfolio?
* Equal Weighting
	+ If something is equally weighted, we can divide the total weight (generally 1) by the number of securities:
* Return of Portfolios
	+ The weighted average of the returns on the investments in the portfolio
	+ These are realized returns (historical)
* Expected return of the portfolio
* Using BA II Plus for rp and E(rp)
	+ Step 1: Data
		- X = returns
		- Y = probabilities (% form)
	+ Step 2: Stat
		- Return =
* Example: Expected Return
	+ Consider 2 securities (stock and bond) whose returns will follow the following distribution:

|  |  |  |  |
| --- | --- | --- | --- |
| Outcome | Probability (ps) | rstock | rbond |
| Boom | 1/3 | 28% | -3% |
| Normal | 1/3 | 12% | 7% |
| Bust | 1/3 | -7% | 17% |

* + What is the expected return on an equally weighted portfolio?
* Volatility of a 2-Stock Portfolio
	+ By combining stocks into a portfolio, we reduce risk
	+ The amount of risk eliminated depends on the degree to which the stocks face common risks and the prices move together
		- To calculate the risk, we need to know the degree to which the stocks’ returns move together
* Covariance and Correlation
	+ Covariance of a portfolio using probabilities
		- The expected product of the derivations of 2 returns from their means
		- Cov(ri,rj)=σij
	+ Estimate of the covariance using historical data
		- Cov(ri,rj)=sij
	+ + (>0): move in the same direction
	+ –(<0): move in the opposite direction
	+ Covariance measures how assets move together but it has a problem:
		- It’s not standardized so we have no way of comparing 2 covariances
		- We may find it useful to have a measure we can compare between pairs of assets
			* The correlation accomplishes this
* Correlation
	+ Calculated by dividing the covariance by the product of the 2 standard deviations
		- This is true whether it is the population or the sample version: only the notation changes
	+ Population:
		- Corr(ri,rj)=ρij
	+ Sample:
		- Corr(ri,rj)=rij
	+ Note: correlation must be between -1 and 1
		- -1: always move oppositely (reduces risk)
		- 0: uncorrelated
		- 1: always move together
* Example: Covariance and Correlation
	+ 2 assets from before

|  |  |  |  |
| --- | --- | --- | --- |
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* + What is the covariance?
	+ Before we can calculate correlation, we need the 2 standard deviations:
	+ What is the correlation?
* Using BA II Plus for sij and rij
	+ Step 1: Data
		- X: first security’s returns
		- Y: second security’s returns
	+ Step 2: Stat
		- Switch to linear regression mode (LIN)
		- r = correlation
		- Sx = standard deviation of first security
		- Sy = standard deviation of second security
		- Covariance = r\*Sx\*Sy
* Example: Covariance and Correlation
	+ You have observed the returns of 2 stocks over the past 4 years:

|  |  |  |
| --- | --- | --- |
| Year | UGA Corp | DGD Holdings |
| 1 | 8% | -5% |
| 2 | 13% | 7% |
| 3 | 2% | 10% |
| 4 | 20% | -3% |

* + What is the covariance and correlation?
* Volatility of a 2-Stock Portfolio
	+ There are two ways variance can be written:
* Example: Volatility of a 2-Stock Portfolio

|  |  |  |  |
| --- | --- | --- | --- |
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* + What is the standard deviation of an equally weighted portfolio?
* Generalizing: volatility of a large portfolio
	+ In general, the variance of a portfolio is equal to the weighted average covariance of each stock with the portfolio
	+ The variance for a portfolio with N securities is:
* Volatility as a Matrix (getting the computer to do it)
	+ Computers store data in matrices (also called vectors when only 1 number wide or tall)
	+ Matrix: set of numbers arranged in a particular way
		- Similar to coordinate system
		- Can be square or rectangular
		- We’ll generally deal with square or single row/column matrices
* Portfolio variance as vector multiplication
	+ Order matters: the first weight is the weight of stock 1, the second is the weight of stock 2, etc.
	+ The covariance of i with j is the same as the covariance of j with i, so σij=σji (σ13= 1st stock and 3rd stock=σ31=3rd stock and 1st stock)
	+ The diagonal of the matrix is the set of variances
* Example: Portfolio Variance in Matrices
	+ Suppose we are given a portfolio consisting of 60% in ABC Corp and 40% in B&O Holdings. ABC’s stock has a variance of .09 while B&O’s variance is .16. The covariance of the two firms is -.01. What is the variance of the portfolio?
* Example: Diversification
	+ Suppose you invest in 2 stocks with the same return. One has a SD of 1 while the other has an SD of 2. They have a covariance of 1/3
	+ Despite the fact that the second stock is inferior, we may still hold it in a portfolio
	+ What is the SD of a portfolio with 75% in the first stock and 25% in the second?
* Takeaways
	+ Weighted average SD = .25 x 2 + .75 x 1 = 1.25
	+ Got a lower risk using the formula than with the weighted average
		- This is true any time that ρ < 1 (risk is lower than weighted average)
		- Risk decreases until, at ρ = -1, we can form a risk-free portfolio
* Mean-Variance Optimization
	+ When choosing the risky assets to include in our portfolio and their weights, we are concerned with both risk and return
	+ Risk-averse investors like more return and less risk so we will compare portfolios based on both expected returns and standard deviations
	+ The mean-variance criterion says that Portfolio A dominates Portfolio B if one of the following is true:
		- E(rA) > E(rB) and σA < σB
		- E(rA) > E(rB) and σA < σB
* Mean-Variance Efficient Frontier
* Minimum-Variance Efficient Frontier
	+ Take every single possible combo of risky assets
	+ Best portfolios have the lowest σ for any given E(r)
		- Global minimum variance is the lowest risk possible
	+ Entire curve on the graph
	+ Best portfolios have the highest E(r) for any given σ
* Similar (But Different) Approaches
	+ We have found a set of efficient portfolios
		- Each has the highest return for a given level of risk
		- Each has the lowest risk for a given level of return
	+ We can’t decide between these two without more information
		- We will add a minimum return
	+ Two approaches:
		- Specify what the minimum return should be
		- Include a risk-free return
* Option 1: Specify a Minimum Return
	+ Investors often have a liquidity or asset value need that makes returns below a certain point highly damaging to the investor
	+ Can specify a minimum acceptable return (rL)
	+ This allows us to define Roy’s Safety-First Criterion
		- Also known as the safety-first ratio
* Interpreting the Safety-First Ratio
	+ Ratio focuses on the portfolio’s shortfall risk
		- Want to choose the portfolio with the highest SF ratio
		- That would be the portfolio that minimizes the probability of returns below the minimum acceptable return
	+ We can measure the probability of falling below the minimum acceptable return as: (not going to calculate)
* Example: SF Ratio
	+ You’re evaluating an investment of $2 million in Troll Capital. You expect the fund to generate a 20% return with a 25% standard deviation. You know that you will need to withdraw $100,000 for personal expenses at the end of the year, and you don’t want to use any money from your initial investment to do so. What is the SF ratio? What is the probability of a shortfall?
	+ Probability of a shortfall would be 27.43%
	+ Takeaway: E(r) = 20% - rL5% = 15% so there is a 15% range in which you are safe, but the SD is 25% which is why there is still a chance of failure
* Caveat: Safety First with a Risk-Free Asset
	+ Purpose of SF is to find an investment with the minimum probability of a shortfall
	+ What if we added a risk-free asset?
		- It would depend on the asset’s return
	+ Rf < rL
		- Nothing changes, the risk-free asset is useless and guarantees a shortfall
	+ Rf > rL
		- No calculations needed: the risk-free asset is optimal
		- Guarantees we avoid a shortfall
* Option 2: Including a Risk-Free Asset
	+ When we allowed ourselves to hold portfolios in addition to individual risk assets, our investment opportunity set increased
		- Resulted in a higher return for a given level of risk
		- At this point, we have all risky assets
	+ We can get the same benefit again by including a risk-free asset
		- Now we can form portfolios that are combinations of a risky portfolio and a risk-free asset
* Before: had the efficient frontier and everything inside was a risky asset
* Generalizing
	+ Your investment opportunity set is the set of all feasible expected return and standard deviation pairs of all portfolios resulting from different weights
	+ Depicted graphically as the Capital Allocation Line (CAL)
		- CAL shows all risk-return combos available to investors
	+ This line will interact with our utility function (from econ), leading us to choose the highest indifference curve given the investment opportunity set
	+ Line will be drawn with Excel
	+ Now you can delete the efficient frontier and just use CAL for portfolios
* Sharpe Ratio
	+ Slope of CAL given by Sharpe Ratio
		- A ratio of reward-to-variability
		- The ratio of excess return of the risky asset to its SD:
	+ This is a risk-adjusted measure
		- Tells us the return per unit of risk
		- Want the highest Sharpe ratio
	+ The best CAL we can achieve is the one with the highest slop, within our investment opportunity set
		- Intersects at a single point
		- Point called the tangency or optimal portfolio
* Efficient Frontier with a Risk-Free Asset
	+ Tangency portfolio > Efficient frontier
	+ Equation of the line:
	+ Note: O subscript means “optimal” for optimal portfolio
* Asset Allocation
	+ Assume you will split your investment funds between safe and risky assets
		- Rf: T-bills or money market fund
		- Rp: Risky portfolio
	+ Return:
		- E(rp)=WrE(rr) + (1-Wr)rf
	+ SD of portfolio
		- SD = Wrσr
* Example: Risk and Return
	+ You manage a risky portfolio with an expected rate of return of 17% and a SD of 27%. The T-bill rate is 7%. Your client chooses to invest 70% in your fund and 30% in a T-bill money market fund. What’s the E(r) and σ of the portfolio?
	+ Given the previous info, what is the reward-to-volatility ratio (S) of your risky portfolio and your client’s overall portfolio?
* CAL: Different Lending and Borrowing Rates
	+ Drawing a straifht line means that we assumed we would be able to borrow and lend at the same rate (the rf rate)
	+ Untrue: we almost always borrow at a higher rate than we are able to lend
	+ We can make CAL more accurate
* Two Fund Separation
	+ Separation property of portfolio theory states that portfolio choice can be divided into 2 independent tasks
		- Identification of the optimal risky portfolio
			* Same for all investors, regardless of risk aversion
			* Represents efficient diversification
		- Allocation choice between the optimal risky portfolio and the risk-free rate
			* Different allocation for investors with different levels of risk aversion
			* All investors will hold some combo of these 2 assets and only these 2 assets