**Risk and Return**

* Portfolio choice
	+ In order to allocate our portfolio, we need to know a few things:
		- We need capital to allocate in the first place
		- We need to specify our goals and preferences
			* Might differ depending on context
			* Not as trivial as immediately appears: e.g. investing for yourself vs. investing for clients
		- We need some way to quantify risk and returns
* Risk and return
	+ Going to be interested in realized returns
	+ Gains from most financial assets come in one of two forms:
		- Capital gains: the difference in the price when we sell vs. when we purchased the asset
		- Cash distributions: cash that is paid to us as an investor
			* Examples are dividends, interest payments, and cash settlements of some derivatives
	+ Single holding period return:
* Example: Holding Period Return
	+ Suppose we purchased a security for $10. While holding it, we receive a payment of $2. Eventually, we sell the security for $12. What is the holding period return?
* Risk and return
	+ Future prices are uncertain
	+ We will define returns as a random variable
		- Random variables don’t have set values
		- There’s a pool of values they can equal
		- The value we observe is a realization of the random variable
			* Statistical way of saying it can change
* More probability
	+ Random variables are drawn from a probability distribution
		- The set of all possible values of a random variable and the probability associated with each possible outcome
	+ Probability represents the chance that a particular outcome is realized
	+ There are 3 key facts about probabilities:
		- Every possible outcome must be assigned a probability
		- Sum of the probabilities must equal 1 or 100%
		- Probability of an event cannot be negative
	+ Can use probability distribution to learn things about the risks and returns of different assets
* Probability distribution
	+ Consider an asset X with a current price of $100. One year from now, this asset will have a payoff. There are 3 states of the world and the assets has a different payoff in each state. Each state is assigned a probability of occurring. The probability distribution of the asset’s payoff is given as:

|  |  |  |  |
| --- | --- | --- | --- |
| Outcome | Probability (ps) | Payoff | Return |
| Boom | 0.2 | $120 | 20% |
| Normal | 0.5 | $110 | 10% |
| Bust | 0.3 | $100 | 0% |

* Expected Return
	+ This table gives us 3 possible outcomes but we want to consolidate to a single number we feel comfortable about
	+ Our best guess of the future return is the expected return
		- Let S be the total number of possible states, and s denotes each possible state where s=1,2,3…S
		- The probability that state s occurs is ps
		- States are mutually exclusive
			* Only one state can occur
			* All possible states are included
		- A risky asset has a rate of return in each of the possible states, denoted as r1,r2,r3…rs
* Expected return formula
	+ E(r) is the probability weighted average return of all possible outcomes
	+ This number usually won’t be correct, but it’s the least wrong on average
		- The misses will average out over time, some high, some low
* Risk Quantified
	+ Since we are generally going to be wrong, we need to deal with that fact
		- We want to know how much the value of the stock varies about that expected return
	+ Variance
		- Expected value of the squared deviation from the mean
* Risk Quantified—Big Question
	+ Why do we use the squared deviation?
		- We want to measure variation from the expected return
		- The square makes every term possible
	+ There are some alternatives: one is the mean absolute deviation, which is computed using absolute values
		- Absolute values can make some of the calculus behind the scenes difficult
		- Squared terms are usually nicer to work with
* Standard Deviation
	+ Using a squared term to measure risk makes certain things unwieldy
		- Hard to compare against expected return in a consistent way
		- We want to convert our risk measure to a linear expression
			* Can use standard deviation for this
	+ Standard deviation: how far returns are from the expected value
* Using BA II Plus for E(r) and σ
	+ Step 1: Data
		- X = returns
		- Y = Probabilities (must use % forms)
	+ Step 2: Stat
		- Make sure screen says 1-V
		- Expected return =
		- Standard deviation = σx
		- Variance = standard deviation squared
* Example: risk and return
	+ Calculate expected return, variance, and standard deviation of asset X using its probability distribution:

|  |  |  |  |
| --- | --- | --- | --- |
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* Empirical Distribution
	+ ~N(μ,σ)
	+ Computing historical returns
		- By counting the number of times a realized return falls within a particular range, we can estimate the underlying probability distribution
		- Often use either the normal or log-normal distributions
	+ Empirical distributions
		- When the probability distribution is plotted using historical data
* Using data
	+ Using data requires us to alter our formulas in some fundamental ways
		- We are no longer assuming we know anything about the future
		- We will be creating values that we hope converge to the true values
		- Our data-driven values are our best estimates of the values we have been calculating
	+ How do we weight the data observations?
		- We generally treat data as if it were equally weighted
		- There’s no reason to believe that a given observation is more significant than another one
* Using data
	+ We’ll replace the expected return with the sample average
		- This is the same average as usual, but presented in formula version
	+ Sample average =
	+ We’ll also need to adjust the variance calculation
		- Using the typical equal weighting (dividing by T) doesn’t actually give us an answer that converges to the true variance
		- We need to divide by T-1
	+ Sample variance =
	+ Standard deviation doesn’t change
* Using the BA II Plus for r and s
	+ Step 1: Data
		- X = returns
		- No need to do anything with the Y terms
	+ Step 2: Stat
		- For average return:
		- For standard deviation: Sx (not σx)
		- Square the standard deviation to find variance if needed
* Example: Using Data
	+ You have collected data on 5 years of returns. You have observed returns of 7%, 8%, -3%, 2%, and 1%. What is the average, variance, and standard deviation of these returns?
* Example: Using Data
	+ Suppose that we are given a set of 3 returns: 4%, 6%, and 11%. What is the average return, variance, and standard deviation of this data?
* Types of Average Returns
	+ Arithmetic
	+ Geometric
	+ Dollar-weighted
* Arithmetic average
	+ The basic sample average we have already used
	+ Answers the question: “What would I expect next period’s returns to be?”
	+ This calculation ignores the effect of compounding
* Geometric average
	+ The constant per period return that yields the same total return over the life of the investment
	+ Answers the question: “What annual return did I effectively earn from this investment?”
* Using BA II Plus for rG
	+ Inputs:
		- PV = -1
		- FV =
		- N = T
		- PMT = 0
		- Compute I/Y
* Example: Averages
	+ A stock yielded 5%, 12%, -2%, and 8% over each of the 4 years. What would you expect next year’s return to be?
	+ Using the same data, what annual return did investors in the stock earn over this period?
* Example: Averages
	+ You purchased a security 3 years ago. Since then, it has generated returns of 6%, 13%, and 2%. What would you predict next year’s return to be?
	+ Using the same data, what annual return is this set of returns equivalent to?
* An important relationship
	+ Because the geometric average incorporates the effects of compounding, its often smaller than we expect
		- We receive interest on interest, so the return needed is lower
	+ Since the arithmetic average ignores compounding, this leads to a consistent relationship:
* Dollar-Weighted Average
	+ The internal rate of return on the investment
	+ Answers the question: “What return sets my discounted cash flows equal to the initial investment?”
	+ Should know this, but won’t calculate it in class
* Thought Experiment: Lottery
	+ Suppose you have a choice of 2 investments:
		- You can pay $1 in exchange for a guaranteed $2 one year from today
		- You can pay $1 today in exchange for an 80% chance of receiving $0 and a 20% chance of receiving $10 one year from today
	+ First option:
	+ Second option:
* Putting it Together
	+ The two choices have the same expected return
		- Choice 2’s SD is higher than choice 1’s SD
* Risk Aversion
	+ Given the choice of 2 investments with equal expected returns, one safe and one risky, there are 3 types of investors:
		- Risk averse: prefers the safe investment
		- Risk neutral: indifferent between the 2
		- Risk loving: prefers the risky investment
	+ We will typically assume that investors are risk averse
* The risk premium
	+ The expected return for making a risky investment rather than a safe one
		- Risk premiums are excess returns
			* Risk premium = E(r) – rf
	+ We need premiums because investors are risk averse
		- They demand extra compensation for investing in assets with risky payoffs
		- If we didn’t have them, all risk averse investors would only invest in T-bills