**Risk and Return**

* Portfolio choice
  + In order to allocate our portfolio, we need to know a few things:
    - We need capital to allocate in the first place
    - We need to specify our goals and preferences
      * Might differ depending on context
      * Not as trivial as immediately appears: e.g. investing for yourself vs. investing for clients
    - We need some way to quantify risk and returns
* Risk and return
  + Going to be interested in realized returns
  + Gains from most financial assets come in one of two forms:
    - Capital gains: the difference in the price when we sell vs. when we purchased the asset
    - Cash distributions: cash that is paid to us as an investor
      * Examples are dividends, interest payments, and cash settlements of some derivatives
  + Single holding period return:
* Example: Holding Period Return
  + Suppose we purchased a security for $10. While holding it, we receive a payment of $2. Eventually, we sell the security for $12. What is the holding period return?
* Risk and return
  + Future prices are uncertain
  + We will define returns as a random variable
    - Random variables don’t have set values
    - There’s a pool of values they can equal
    - The value we observe is a realization of the random variable
      * Statistical way of saying it can change
* More probability
  + Random variables are drawn from a probability distribution
    - The set of all possible values of a random variable and the probability associated with each possible outcome
  + Probability represents the chance that a particular outcome is realized
  + There are 3 key facts about probabilities:
    - Every possible outcome must be assigned a probability
    - Sum of the probabilities must equal 1 or 100%
    - Probability of an event cannot be negative
  + Can use probability distribution to learn things about the risks and returns of different assets
* Probability distribution
  + Consider an asset X with a current price of $100. One year from now, this asset will have a payoff. There are 3 states of the world and the assets has a different payoff in each state. Each state is assigned a probability of occurring. The probability distribution of the asset’s payoff is given as:

|  |  |  |  |
| --- | --- | --- | --- |
| Outcome | Probability (ps) | Payoff | Return |
| Boom | 0.2 | $120 | 20% |
| Normal | 0.5 | $110 | 10% |
| Bust | 0.3 | $100 | 0% |

* Expected Return
  + This table gives us 3 possible outcomes but we want to consolidate to a single number we feel comfortable about
  + Our best guess of the future return is the expected return
    - Let S be the total number of possible states, and s denotes each possible state where s=1,2,3…S
    - The probability that state s occurs is ps
    - States are mutually exclusive
      * Only one state can occur
      * All possible states are included
    - A risky asset has a rate of return in each of the possible states, denoted as r1,r2,r3…rs
* Expected return formula
  + E(r) is the probability weighted average return of all possible outcomes
  + This number usually won’t be correct, but it’s the least wrong on average
    - The misses will average out over time, some high, some low
* Risk Quantified
  + Since we are generally going to be wrong, we need to deal with that fact
    - We want to know how much the value of the stock varies about that expected return
  + Variance
    - Expected value of the squared deviation from the mean
* Risk Quantified—Big Question
  + Why do we use the squared deviation?
    - We want to measure variation from the expected return
    - The square makes every term possible
  + There are some alternatives: one is the mean absolute deviation, which is computed using absolute values
    - Absolute values can make some of the calculus behind the scenes difficult
    - Squared terms are usually nicer to work with
* Standard Deviation
  + Using a squared term to measure risk makes certain things unwieldy
    - Hard to compare against expected return in a consistent way
    - We want to convert our risk measure to a linear expression
      * Can use standard deviation for this
  + Standard deviation: how far returns are from the expected value
* Using BA II Plus for E(r) and σ
  + Step 1: Data
    - X = returns
    - Y = Probabilities (must use % forms)
  + Step 2: Stat
    - Make sure screen says 1-V
    - Expected return =
    - Standard deviation = σx
    - Variance = standard deviation squared
* Example: risk and return
  + Calculate expected return, variance, and standard deviation of asset X using its probability distribution:

|  |  |  |  |
| --- | --- | --- | --- |
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* Empirical Distribution
  + ~N(μ,σ)
  + Computing historical returns
    - By counting the number of times a realized return falls within a particular range, we can estimate the underlying probability distribution
    - Often use either the normal or log-normal distributions
  + Empirical distributions
    - When the probability distribution is plotted using historical data
* Using data
  + Using data requires us to alter our formulas in some fundamental ways
    - We are no longer assuming we know anything about the future
    - We will be creating values that we hope converge to the true values
    - Our data-driven values are our best estimates of the values we have been calculating
  + How do we weight the data observations?
    - We generally treat data as if it were equally weighted
    - There’s no reason to believe that a given observation is more significant than another one
* Using data
  + We’ll replace the expected return with the sample average
    - This is the same average as usual, but presented in formula version
  + Sample average =
  + We’ll also need to adjust the variance calculation
    - Using the typical equal weighting (dividing by T) doesn’t actually give us an answer that converges to the true variance
    - We need to divide by T-1
  + Sample variance =
  + Standard deviation doesn’t change
* Using the BA II Plus for r and s
  + Step 1: Data
    - X = returns
    - No need to do anything with the Y terms
  + Step 2: Stat
    - For average return:
    - For standard deviation: Sx (not σx)
    - Square the standard deviation to find variance if needed
* Example: Using Data
  + You have collected data on 5 years of returns. You have observed returns of 7%, 8%, -3%, 2%, and 1%. What is the average, variance, and standard deviation of these returns?
* Example: Using Data
  + Suppose that we are given a set of 3 returns: 4%, 6%, and 11%. What is the average return, variance, and standard deviation of this data?
* Types of Average Returns
  + Arithmetic
  + Geometric
  + Dollar-weighted
* Arithmetic average
  + The basic sample average we have already used
  + Answers the question: “What would I expect next period’s returns to be?”
  + This calculation ignores the effect of compounding
* Geometric average
  + The constant per period return that yields the same total return over the life of the investment
  + Answers the question: “What annual return did I effectively earn from this investment?”
* Using BA II Plus for rG
  + Inputs:
    - PV = -1
    - FV =
    - N = T
    - PMT = 0
    - Compute I/Y
* Example: Averages
  + A stock yielded 5%, 12%, -2%, and 8% over each of the 4 years. What would you expect next year’s return to be?
  + Using the same data, what annual return did investors in the stock earn over this period?
* Example: Averages
  + You purchased a security 3 years ago. Since then, it has generated returns of 6%, 13%, and 2%. What would you predict next year’s return to be?
  + Using the same data, what annual return is this set of returns equivalent to?
* An important relationship
  + Because the geometric average incorporates the effects of compounding, its often smaller than we expect
    - We receive interest on interest, so the return needed is lower
  + Since the arithmetic average ignores compounding, this leads to a consistent relationship:
* Dollar-Weighted Average
  + The internal rate of return on the investment
  + Answers the question: “What return sets my discounted cash flows equal to the initial investment?”
  + Should know this, but won’t calculate it in class
* Thought Experiment: Lottery
  + Suppose you have a choice of 2 investments:
    - You can pay $1 in exchange for a guaranteed $2 one year from today
    - You can pay $1 today in exchange for an 80% chance of receiving $0 and a 20% chance of receiving $10 one year from today
  + First option:
  + Second option:
* Putting it Together
  + The two choices have the same expected return
    - Choice 2’s SD is higher than choice 1’s SD
* Risk Aversion
  + Given the choice of 2 investments with equal expected returns, one safe and one risky, there are 3 types of investors:
    - Risk averse: prefers the safe investment
    - Risk neutral: indifferent between the 2
    - Risk loving: prefers the risky investment
  + We will typically assume that investors are risk averse
* The risk premium
  + The expected return for making a risky investment rather than a safe one
    - Risk premiums are excess returns
      * Risk premium = E(r) – rf
  + We need premiums because investors are risk averse
    - They demand extra compensation for investing in assets with risky payoffs
    - If we didn’t have them, all risk averse investors would only invest in T-bills