* Extending the binomial model
  + As we increase the number of branches in the binomial model, each branch represents a smaller period of time
    - This refinement gives us a more realistic picture of the way stock prices move over time
  + As we are able to rebalance our replicating portfolio, we can refine our estimate of the value of the call
  + We can show that as the number of branches increases, the value of the option will oscillate around its limit
  + In fact, we can show that the binomial model converges to a particular price as the number of branches goes to infinity
    - This is the Black-Scholes price of the option
* The Black-Scholes Formula
  + Black and Scholes (1973) and Merton (1973) were able to derive a precise option pricing formula
  + The Black-Scholes formula says that the price of a call is:
  + Then, using put-call parity, the price of a put is:
* Notes to the Black-Scholes Formula
  + S is the current stock price
  + X is the exercise price
    - if interest rates are continuously compounded
  + N(d) is the cumulative normal distribution function
  + σ is the annual volatility of the stock
  + T is the time remaining until expiration **in years**
* Cumulative Normal Distribution Function
  + The probability that an outcome from a standard normal distribution will be below a certain value
    - Represented by the area to the left of the input value
    - Generally found using Excel or referring to a table
* Example: Black-Scholes—d1
  + Assume:
    - CLW does not pay dividends
    - The standard deviation of CLW is 45% per year
    - The risk free rate is 5%
    - CLW has a current price of $24
  + Using the Black-Scholes formula, what is the price for a 6 month American call option on CLW with a strike price of $30? A put option?
* Example: Black-Scholes—d2, N(d)
* Example: Black-Scholes—PV(X)
* Example: Black-Scholes—Putting it Together
* Black-Scholes Formula Inputs
  + Despite all math, only five inputs are needed for the formula. They are:
    - Stock price
    - Strike price
    - Exercise date
    - Risk-free rate
    - Volatility of the stock
  + The first three are given in the option contract, and the risk-free rate can be found using the Treasury. So, all of these but the stock’s volatility can be relatively easy and accurately observed
* Implied Volatility
  + Of the five required inputs in the Black-Scholes formula, only σ is not observable directly
  + Practitioners generally use two strategies to estimate the value of σ:
    - Use historical data
    - “Back out” the implied volatility
  + Implied volatility is the volatility of an asset’s returns that is consistent with the quoted price of an option on the asset
* Black-Scholes and the Replicating Portfolio
  + Given the option price in the binomial model:
  + The Black-Scholes replicating portfolio of a call option is:
  + Recall our interpretations of from the binomial model:
    - The number of shares in the replicating portfolio for the option
    - The change in price of an option given a $1 change in the price of the stock
  + These are still true, but now we are talking about a constantly changing portfolio
    - We know the number of shares at any given instant, but it changes just as quickly
    - It would be effectively impossible to actually invest in the replicating portfolio under continuous time
    - This doesn’t mean that the idea is useless though
* Risk and the Replicating Portfolio
  + We can use the replicating portfolio to learn about the risk of an option
  + The replicating portfolio of a call option will always consist of a long position in the stock and a short position in the bond
    - The replicating portfolio is a leveraged position in the stock
  + It’s tempting to think of an option as a safer investment than a stock
    - It requires a smaller investment, meaning that we generally have less money at risk
  + However, the smaller upfront cost means that the returns on an option are magnified
    - Even small changes in its value can lead to large swings in returns
* Option Beta
  + Consider that, for a given share price, we can purchase a single share or multiple option contracts
    - Each option’s price would change by for each $1 movement in the stock’s price
  + With this in mind, the beta of an option is:
    - βCβS
    - βPβS
* Example: option betas
  + A share in XY Corp. currently sells for $60. Meanwhile, a six month put option is available for $6. Suppose that the put option would be worth $4 if the stock went up to $66 and $12 if the stock went down to $50. If you have estimated the beta of XY Corp.’s stock to be 1.25, what is the beta of this option?