**Option Valuation**

* Decomposing option values
  + Intrinsic value
    - The amount by which the option is in the money
    - 0 if the option is out of the money
  + Time value
    - The difference between the option’s price and the intrinsic value
    - Reflects the fact that there is still time before expiration
      * C = IV + TV
* Arbitrage bounds
  + A call option cannot be worth more than the stock itself (soft limit)
    - Max of C = max(S-$0, $0) = max(S, $0) = S
  + A put option cannot be worth more than its strike price (hard limit)
    - Max of P = max(X-$0, $0) = max(X, $0) = X
  + An American option cannot be less than its European counterpart
    - Allows for the same usage on the expiration date, and more
* Factors affecting option prices
  + 6 primary influences—4 on intrinsic value, 2 on time value
  + Affect intrinsic value:
    - Stock price
    - Strike price
    - Interest rate
    - Dividend payout
  + Affect time value:
    - Volatility
    - Time to expiration
* Factors affecting intrinsic values
  + Stock price
    - As the underlying stock price increases, the value of a call option increases
      * Your expense is locked in, while the value of the share is increasing
    - As the underlying stock price increases, the value of a put option decreases
      * Your income is locked in, while the value of the share is increasing
  + Strike price
    - As the exercise price increases, the value of a call option decreases
      * Your expense is rising, while the value of the share remains constant
    - As the exercise price increases, the value of a put option increases
      * Your income increases, while the value of the share remains constant
  + Interest rate
    - As the market interest rates increase, the value of a call option increases
      * Devalues the risk free bond, requiring the call value to increase to maintain parity
    - As market rates increase, the value of a put option decreases
      * Devalues the risk free bond, requiring the put value to decrease to maintain parity
  + Dividend payout
    - As the firm’s dividend payout increases, the value of a call option decreases
      * Dividends represent cash leaving the firm, which should result in a lower firm value
    - As the firm’s dividend payout increases, the value of a put option increases
      * Dividends represent cash leaving the firm, which should result in a lower firm value
* Factors affecting time values
  + Volatility
    - Increased volatility increases the value of all put and call options
      * Optionholders can exercise to take advantage of volatility, increasing the value of the option
      * Optionholders can allow the option to expire if the volatility results in the option being out of the money
  + Time to expiration
    - A longer time to expiration almost always increases an option’s value
      * Allows more time for the option to increase in value
      * Can actually decrease the value of an in the money European put
        + Has a limited upside and cannot mitigate losses by exercising early
* Binomial option pricing
  + How can we value a derivative relative to its underlying asset?
  + Can the payoffs of a derivative asset be replicated by trading only in the underlying asset (and possibly cash)?
    - If we can find such a replicating strategy, the current value of the option must equal the initial cost of the replicating portfolio
    - This allows us to create a non-existent derivative by following its replicating strategy
  + This is the central idea behind the modern option pricing theory
* Two states in a single period model
  + Consider a European call option that expires in one year and has an exercise price of $50. Currently, the price of the underlying stock is $50, and the stock pays no dividends. Suppose that over the next year the stock price will either rise by $10 or fall by $10. The one year interest rate is 6%
    - Next year, the stock will be worth $40 if it decreases in value
    - Next year, the stock will be worth $60 if it increases in value
    - Next year, a $1 investment in the risk free bond will be worth $1.06 regardless of what happens with the stock
* Replicating portfolio
  + Can we find a portfolio of the stock and the bond that has the same payoffs as the call?
    - Yes, by:
      * Construct a portfolio that has Δ shares of the underlying stock and B of the risk free asset
      * Set the payoff of this portfolio equal to the payoff of the option in each state
      * Solve for Δ and B
* Example: Replicating Portfolio
  + Consider a European call option that expires in one year and has an exercise price of $50. Currently, the price of the underlying stock is $50 and the stock pays no dividends. Suppose that over the next year, the stock price will either rise by $10 or fall by $10. The one year interest rate is 6%
  + The value of the portfolio today is the value of 0.5 shares at the current share price of $50, less the amount borrowed:
  + By the law of one price, the price of the call option must equal the market value of the replicating portfolio
  + Note that by using the law of one price, we are able to solve for the price of the option without knowing the probabilities of the outcomes
* Binomial option pricing formula
  + Assume
    - S is the current stock price and S will either go up to Su or down to Sd next period
    - Cu id the value of the call option if the stock price goes up and Cd is the value of the call option if the stock goes down
  + Solving the 2 year replicating portfolio equations for the two unknowns Δ and B yields the general formula for the replicating portfolio in the binomial model:
* Understanding the replicating portfolio
  + How can we interpret the option delta Δ?
    - The number of shares in the replicating portfolio for the option
    - The change in price of an option given a $1 change in the price of the stock
    - The number of shares that can be hedged using one option
      * This interpretation causes the delta to be known as the hedge ratio (important in risk management)
  + Note that because Δ is always less than or equal to 1, the change in call price is always less than or equal to the change in the stock price
* Example: Binomial Put Pricing
  + A stock is currently trading at $60 and in one period will either go up by 20% or down by 10%. If the one period risk free rate is 3%, what is the price of a European put option that expires in one period and has an exercise price of $60?
* Alternative View: the two state model
  + The binomial model is set up using the idea of a replicating portfolio
    - This is not necessarily the only interpretation
  + Using the binomial model, we replicate the option using the stock and a risk-free bond
    - What if we used the option and the stock in order to replicate the risk-free bond?
    - We could form a risk free portfolio and use that to price the option
  + Note: these two approaches will yield the same answer because we are ultimately performing the same calculations, we are just performing them differently
* Using the two state model
  + Under a two state model, we would still calculate Δ as normal
  + From there, things change
  + Next, we take a reciprocal to find the number of options hedged by 1 share of the underlying stock
    - We use this to find a risk free cash flow
  + We set up an equation between the stock, the option(s), and the risk free cash flow and solve for the value of the option(s)
* Example: two state model
  + A stock is currently priced at $60 and it may rise by 20% or fall by 10%. The risk free rate is 3% and we want the price of the put option with an exercise price of $60.
* Extending the binomial model
  + In modeling the stock prices as binomial, we are making a very strong simplifying assumption that the future stock price will be one of two values
    - But we know that there are more than two states of the world
  + The binomial model does not have to limit us to just two future states
    - By extending our two state model to represent smaller periods of time, we can generate more future states, making the model more realistic
* Multi-period model
  + We can extend the binomial model to two periods or more
    - Each node creates a new tree for the next period
  + Notice that the price at each node is determined by the previous price adjusted by whether you moved up or down
* Multi-period model process
  + Step 1: calculate the prices for all nodes
    - Start with today’s price and move to the right
  + Step 2: calculate option payoffs at expiration
    - Use payoff formulas from the last chapter (e.g. C = max(S-X, $0))
    - Important: we will assume this option is European so only do this in the final period
  + Step 3: calculate a binomial tree for each node
    - Starting with the final period, work to the left
* Multiperiod Model
  + Consider a two-period binomial tree for the stock price. You are interested in a call option, with an exercise price of $50. Suppose that the one year risk free rate is 12% (so, 6% each 6 months).
  + If we are modeling the stock price over one year, for example, then each branch represents six months
* Solving the multiperiod model
  + Suppose that the stock price rises to $50 at time 1
    - At time 2, the option expires, so its value is equal to its intrinsic value
    - The call will be worth $10 if the stock price goes up again to $60
    - The call will be worth $0 if the stock price goes down to $40
    - Δ=.5 and B=-$18.87, so the value of the call at time 1 will be $6.13
  + Consider the second case, where the stock price falls to $30 at time 1
  + Now that we know the value of the option in each possible state at time 1, we can find the value of the option today
* Dynamic Trading Strategy
  + We can replicate the option payoff by dynamically trading in a portfolio of the underlying stock and a risk-free bond
    - We adjust the replicating portfolio at the end of each period
  + In the two-period example before, the portfolio starts off long 0.3065 shares of stock and borrowing $8.67