

Normal Distribution Formulae:

Sample Mean =

n = number of observations

$$\text{Sample Variance} = s^2 = \frac{\sum(x-\bar{x})^2}{n}$$

$$\text{Shortcut} = s^2 = \frac{\sum x^2 - \left(\frac{\sum x}{n}\right)^2}{n}$$

$$\text{Sample Standard Deviation} = s = \sqrt{s^2} = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

Stock Return Formulae:

E(x) = expected return of stock x

$$\text{Average Return} = \frac{E(x)+E(y)}{2}$$

$$\text{Average Return for Multiple Stocks} = \frac{E(x_1)+\dots+E(x_n)}{n}$$

n = number of stocks

Variance of 2 correlated Stocks –

$$\text{var}(x + y) = \text{var}(x) + \text{var}(y) + 2 \text{cov}(x, y)$$

$$\text{Standard Deviation} = \sqrt{\text{var}(x, y)}$$

I.I.D. Formulae:

Standard Deviation: Recall variance and standard deviation formulae from earlier (normal distribution)

To find the standard deviation you must first find the variance

Variance: For any two random variables, that are uncorrelated: $\text{Cov}(x,y) = 0$

$$\text{var}(x + y) = \text{var}(x) + \text{var}(y)$$

$$\text{Standard Deviation} = \sqrt{\text{var}(x) + \text{var}(y)} \text{ or } \sqrt{\text{var}(X + Y)}$$

I.I.D. Average of Sums: Variance

2 Stock Portfolio:

$$\begin{aligned} \text{var}\left(\frac{x + y}{2}\right) &= \text{var}\left(\frac{x}{2}\right) + \text{var}\left(\frac{y}{2}\right) \\ &= \left(\frac{1}{2}\right)^2 [\text{var}(x) + \text{var}(y)] = \frac{1}{4} [\text{var}(x) + \text{var}(y)] \end{aligned}$$

Multi Stock Portfolio:

$$\left(\frac{1}{n}\right)^2 [\text{var}(x_1) + \text{var}(x_2) + \dots + \text{var}(x_n)]$$

I.I.D. Average of Sums: Standard Deviation

σ_p = Standard Deviation of portfolio:

$$\begin{aligned}\sigma_p &= \sqrt{\text{var}\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right)} \\ &= \sqrt{\left(\frac{1}{n}\right)^2 \text{var}(x_1 + x_2 + \dots + x_n)} \\ &= \sigma_p = \frac{1}{n} \sqrt{\text{var}(x_1) + \text{var}(x_2) + \dots + \text{var}(x_n)}\end{aligned}$$

Note: if $\text{var}(x_1) = \text{var}(x_2) = \text{var}(x_n) = \text{var}(x)$

$$\sigma_p = \frac{1}{n} \sqrt{n \cdot \text{var}(x)}$$

$$\sigma_p = \frac{\sqrt{n}}{n} \cdot \sqrt{\text{var}(x)}$$

$$\sigma_p = \frac{\sigma(x)}{\sqrt{n}}$$

Note: $n = (\sqrt{n})^2$

$$\sigma(x) = \sqrt{\text{var}(x)}$$