**Asset Pricing**

* Optimal Portfolio Choice—Simple Proof
	+ Call the tangency portfolio M
	+ Suppose M did not contain MSFT
		- Everyone should hold M and no one should hold MSFT
		- M’s price (and the prices of the stocks inside M) would increase
		- MSFT’s price would decrease
	+ In equilibrium, we would ultimately hold M and MSFT
		- MSFT’s price would decrease until its return was high enough to be include in M
		- While M’s price increased, return decreased
		- While MSFT’s price decreased, return increased, and eventually it will become attractive enough to be included in M
	+ We could repeat the example with any asset not included in M
		- Ultimately all of these assets would be included
	+ M on previous slide must be the market portfolio
		- Contains all risky assets in the economy
		- Value-weighted
* Capital Market Line (CML)
	+ A specific CAL in which the risk-free rate is given by a T-bill and the optimal risky portfolio is given by a broad-based index of common stocks
		- Index often known as the “market portfolio”
		- In theory, CML has the steepest slope of all possible CALs
* Problems with CML
	+ No way to actually hold the market portfolio
	+ We will always miss some asset
	+ So CML is effectively unachievable, but the idea still helps to model broad market behavior
* CAPM
	+ Equilibrium model of security markets that attempts to answer the question: “What is the relationship between risk and expected returns?”
	+ It is a single factor asset pricing model:
		- Describes the risk-return relationship for individual assets
		- An asset’s risk is characterized by its contribution to the risk of an efficient, diversified portfolio
		- The single factor in the market portfolio, and an asset’s relevant risk is its market (systematic) risk
* CAPM Assumptions
	+ Investors are rational mean-variance optimizers
		- They form portfolios based on mean returns and variances
	+ Homogenous expectations
		- All investors have the same probability distribution of future events
	+ All investors can borrow and lend at rf rate
	+ Markets are perfectly competitive
	+ Investors have a single-period investment horizon
	+ No market frictions
		- No taxes
		- No commissions, bid-ask spread, etc.
* CAPM Equilibrium
	+ Under CAPM, all investors have the same opportunity set
		- Means they’re using the same method to evaluate the same investments
		- They will invest in the same assets
	+ Under these conditions, equilibrium outcome is:
		- All investors hold the same portfolio (M)
		- Market portfolio (M) is the optimal risky portfolio
		- The CAL that runs through M from the risk-free rate is the best CAL
			* This is the CML
* Portfolio M and Expected Returns
	+ These results imply that the relevant risk of a particular stock is the risk that it adds to the market portfolio
	+ Consider adding a little more of stock i to the market portfolio, borrowing at rate rf to pay for it
		- The change in E(r) is proportional to (ri-rf)
		- The change in standard deviation is proportional to σm
		- This implies that Δreturn/Δrisk is proportional to (ri-rf)/ σm
* Deriving CAPM
* CAPM
	+ Describes the expected return on a risky asset as a function of:
		- The risk-free rate
		- The expected market risk premium
		- The systematic risk of the asset
* Example: CAPM
	+ Beta for Wells Fargo is 1.1. Expected market risk premium is 7.2% and rf is 1%. What is the expected return for Wells Fargo?
* Security Market Line (SML)
	+ Describes the equilibrium relationship between the systematic risk of any individual asset (or portfolio) and its expected return
	+ It is the expected return-beta relationship for any asset or portfolio since beta is our measure of systematic risk
* Example: SML
	+ Consider the following info for the market portfolio (M) and 2 risky assets, A and B:

|  |  |  |
| --- | --- | --- |
| E(rm) = 8% | σm = 16% | Rf = 3% |
| E(rA) = 12% | σA = 36% | σAM = 460.8 |
| E(rB) = 5% | σB = 10.667% | σBM = 102.4 |

* + Calculate the Betas for A and B
* Mispricing and Abnormal Returns
	+ CAPM is an equilibrium model
		- Actual realized returns will often differ from what was predicted
		- Results in mispricing relative to CAPM
		- We can measure mispricing, or abnormal return with alpha:
		- α > 0: positive abnormal return, security is underpriced
		- α < 0: negative abnormal return, security is overpriced
			* Ri = actual return
			* Subtract out CAPM
			* If return is too low, price is too high, and vice versa
			* Want to find positive alphas and avoid negative alphas
* Alpha and the SML
* Example: Mispricing
	+ You are evaluating 2 securities to add to your portfolio
		- Stock A has a beta of 0.85 and an average historical return of 11.5%
		- Stock B has a beta of 1.3 and an average historical return of 14%
	+ The expected market risk premium is 9.2% and the risk-free rate is 3%. You believe equilibrium rates of return are completely described by CAPM. Based on each security’s alpha, which should you buy?
* CAPM Application #1
	+ Investment Performance Evaluation
		- CAPM can provide a benchmark return
			* Based on beta of the portfolio being managed
		- Can compare to the actual, realized return
			* If the manager has exceeded the benchmark, then α > 0
			* If the manager has underperformed the benchmark, then α < 0
		- Keep in mind: CAPM is not the only benchmark available
* CAPM Application #2
	+ Capital Budgeting
		- CAPM can provide the required rate of return (hurdle rate) for a capital project, based on the project’s beta
* Estimating CAPM
	+ Gives us a function for expected returns:
	+ In order to estimate CAPM beta, we want to figure out the correlation between the market risk premium and the expected return
		- The rf will change over time but these changes will have nothing to do with the stock or the market risk
		- We will subtract out rf and focus on the market return
* Estimating CAPM
	+ To estimate beta, we use historical returns as our proxy for expected returns
	+ We estimate beta by running a regression of excess stock returns on the excess returns of the market portfolio:
	+ Note:
		- We are regressing excess returns (risk premium)
		- Ei,t is the error term and is estimated to be 0
* Estimating CAPM
	+ Estimating the regression above gives us the security characteristic line (SCL)
		- The slope of this line is estimated by the beta coefficient:
		- The intercept of this line is αi
* SCL Illustrated
* Estimating Beta: Data Notes
	+ We are using realized returns as a proxy for expected returns
		- Assuming that you can use past performance to predict future performance
	+ Two typical proxies for the market portfolio are:
		- S&P 500
		- A value-weighted index of stocks on the NYSE, Amex, and Nasdaq
	+ Both proxies ultimately focus on large US stocks
* Testing CAPM
	+ CAPM represents one of the many asset pricing models
		- We can test how well it’s predictions hold up using pricing models
	+ Recall the regression equation:
	+ In a CAPM world, on average we would expect to see:
		- αi = 0
		- Betai = 1
* Using the BA II Plus for α and Beta
	+ Data
		- X = market risk premium for each period
		- Y = security’s excess returns
	+ Stat
		- LIN
		- a = α
		- b = beta
* Example: CAPM Regression
	+ You have gathered some data on the firm’s returns over the last 3 years. It is presented in the following table:

|  |  |  |  |
| --- | --- | --- | --- |
| Year | RegCo | S&P 500 | T-Bill Rate |
| 1 | 10% | 12% | 1% |
| 2 | 13% | 14% | 1% |
| 3 | 6% | 9% | 2% |

* + What is the firm’s alpha and beta over this time period?
* CAPM Empirical Evidence
	+ Empirical tests of CAPM have documented several issues in using CAPM to explain average returns:
		- Large stocks earn returns lower than predicted
		- Small stocks earn returns higher than predicted
		- Growth stocks earn lower returns than predicted
		- Value stocks earn higher returns than predicted
	+ CAPM says that systematic risk is the only thing that matters for an asset’s return, so we shouldn’t be able to find systematic trends like this
* Interpreting the evidence
	+ Mispricing argument
		- Large stocks are overpriced relative to small stocks
		- Growth stocks are overpriced relative to value stocks
	+ Risk premium argument
		- Small stocks have extra risks in relation to large stocks
		- Value stocks have extra risks in relation to growth stocks
			* In both cases, risks are not captured by beta, so investors will demand a risk premium for owning these
* Multi-Factor Models
	+ CAPM is a single-factor asset pricing model
		- The factor is the excess return on the market portfolio
		- The empirical evidence suggests this factor alone does not fully explain asset returns
		- There might be other risk factors that influence returns
	+ Multi-factor asset pricing models may do a better job, because investors may demand additional risk premiums for additional risk factors
* Fama-French 3 Factor Model
	+ Recall the evidence that:
		- Small size stocks outperform large size stocks
		- High book-to-market (value) stocks outperform low book-to-market (growth) stocks
	+ Fama and French proposed the following model:
	+ Market Factor (RMRF or rm-rf)
		- Market risk premium
	+ Size Factor (SMB)
		- The return difference between a portfolio of small stocks and a portfolio of large stocks (small minus big)
		- RSMB = rsmall – rbig
	+ Book-to-Market Factor (HML)
		- Return difference between a portfolio of high book-to-market stocks and a portfolio of low book-to-market stocks (high minus low)
		- RHML = rhighBM – rlowBM
* Fama-French 3 Factor Model
	+ The factors are the excess returns (or risk premiums) to what are known as factor mimicking portfolios
		- We can’t actually observe the factors, so we built these portfolios to estimate what we think the factor’s value is
		- The mimicking portfolio returns are the same stocks, so we want to set up a model like:
		- The factors loading, or betas, represent each stock’s sensitivity to the factor
		- We may view the additional (SMB and HML) factors as risk premiums similar to the market risk premium
			* Interpret SMB as the risk-premium demanded by investors for taking on the additional risk of holding small firms relative to large firms
			* Interpret HML as the risk-premium demanded by investors for taking on the risk of owning high book-to-market stocks
		- Whether these additional factors measure actual sources of fundamental risk is still a matter of debate
		- Empirically, these factors do help explain average returns
		- There are some potential justifications:
			* Maybe book-to-market is a proxy for financial distress risk
			* High book-to-market stocks (e.g. firms with high book value of equity relative to market value of equity) may potentially be near financial distress
* Using the 3 Factor Model
	+ One use for the model would be to tease out the abnormal return (the intercept when we run a regression)
	+ Our regression model for investment i would look like:
* Example: FF 3 Factor Model
	+ You expect that the risk premia for the 3 FF factors will be:
		- Rm-rf = 6%
		- RSMB = 8%
		- RHML = 3%
	+ You regress excess returns for stocks A and B on the 3 factors to estimate each stock’s factor loading (beta) for each of the 3 factors
	+ These regressions give the following results

|  |  |  |  |
| --- | --- | --- | --- |
| Stock A: α = 2% | βRMRF = 1.2 | βSMB = -0.5 | βHML = 2 |
| Stock B: α = -0.5% | βRMRF = 0.75 | βSMB = 2 | βHML = 0.5 |

* + If the risk-free rate is 2%, what is the excess return for each stock?
* Arbitrage Pricing Theory
	+ Gives a relationship for expected returns that relies on “no arbitrage” requirements
		- It prices securities based on the notion that arbitrage opportunities do not exist in well-functioning capital markets
	+ Mispricings can occur
		- Investors are able to profit from these through trading
		- Investor finds two stocks with equivalent risk
		- Investor shorts the lower return and buys the higher return
		- Arbitrage: simultaneous purchase and sale of an asset to profit from the difference in price, results from market inefficiencies
			* Ex. Stock is trading on NYSE for $20 but on London Exchange for $20.05, so you could buy on NYSE and sell on London, making a $0.05 profit
* Arbitrage clarifications
	+ A riskless arbitrage opportunity:
		- Occurs when positive payoffs are realized with certainty with zero upfront investment
		- A “zero-investment” portfolio can be formed by shorting the overpriced security and buying the underpriced security
	+ The clearest version of a riskless arbitrage is a violation of the law of one price when the same asset sells for different prices in two different markets
* APT Assumptions
	+ All securities have finite returns and variances
	+ Some people can form well-diversified portfolios
	+ No taxes or transaction costs
* CAPM vs. APT

|  |  |
| --- | --- |
| **CAPM** | **APT** |
| Equilibrium model | No arbitrage model |
| Single factor completely describes expected returns | Allows for multiple risk factors to determine expected returns(doesn’t specify what they are) |
| Many restrictive assumptions(including the existence of an unobservable market portfolio) | Fewer assumptions(no market portfolio) |
| Applies to individual assets or portfolios | Applies to well-diversified portfolios |

* Multifactor APT
	+ A generalized version of APT with multiple factors describes the expected return of a well-diversified portfolio
	+ For example, a model with k factors is given as:
	+ Where:
		- βp1…βpk = the factor sensitivities to portfolio p
		- λ1…λk = the risk premiums for each factor
		- λ0 = a constant equal to the risk-free rate, if one exists
	+ We could write the model in a more familiar form:
* APT Factors
	+ One of the benefits of APT is that we aren’t restricted to any particular factors
	+ There are, however, some suggested guidelines for factors:
		- Their impact on asset prices manifests in their unexpected movements
		- They represent undiversifiable influences (typically macroeconomic)
		- Timely and accurate info on these variables is required
		- Their relationship should be theoretically justifiable on economic grounds
* Example ATP Factors
	+ Chen, Ross, and Roll (1986) used the following:
		- Surprises in inflation
		- Surprises in GNP
		- Surprises in investor confidence (measured by changes in the default premium of corporate bonds)
		- Surprise shifts in the yield curves
* Notes About APT Factors
	+ One of the problems with the factors mentioned in the last slide is that they are “slow”
		- They may be reported monthly or even quarterly, which isn’t necessarily fast enough for today’s market
	+ People often use derivative prices in place of these factors in order to have more “active” data
* APT: Factor Mimicking Portfolios
	+ We often need to create portfolios whose returns are perfectly positively correlated with the underlying factors, especially if the factor is not a traded asset
		- There are factor mimicking portfolios, which are construed similarly to the 3-factor model
			* Unlike the Fama French mimicking portfolios, we’ll only use a long position bearing the risk
		- The risk premium on the mimicking portfolio serves as a proxy for the factor risk premium:
			* λ1 = E(r1) – rf
			* Where r1 is the return on the mimicking portfolio for factor 1
	+ Using this, we can rewrite the general version of the APT using the excess returns of factor mimicking portfolios:
	+ We can use as many or whatever factors we want to, with 2 caveats:
		- We must use 1 less factor than the number of assets we have data on
		- We should also avoid the temptation to “p-hack”
			* Stick to variables that “mostly” fit the guidelines
			* Data mining can be dangerous