**Asset Pricing**

* Optimal Portfolio Choice—Simple Proof
  + Call the tangency portfolio M
  + Suppose M did not contain MSFT
    - Everyone should hold M and no one should hold MSFT
    - M’s price (and the prices of the stocks inside M) would increase
    - MSFT’s price would decrease
  + In equilibrium, we would ultimately hold M and MSFT
    - MSFT’s price would decrease until its return was high enough to be include in M
    - While M’s price increased, return decreased
    - While MSFT’s price decreased, return increased, and eventually it will become attractive enough to be included in M
  + We could repeat the example with any asset not included in M
    - Ultimately all of these assets would be included
  + M on previous slide must be the market portfolio
    - Contains all risky assets in the economy
    - Value-weighted
* Capital Market Line (CML)
  + A specific CAL in which the risk-free rate is given by a T-bill and the optimal risky portfolio is given by a broad-based index of common stocks
    - Index often known as the “market portfolio”
    - In theory, CML has the steepest slope of all possible CALs
* Problems with CML
  + No way to actually hold the market portfolio
  + We will always miss some asset
  + So CML is effectively unachievable, but the idea still helps to model broad market behavior
* CAPM
  + Equilibrium model of security markets that attempts to answer the question: “What is the relationship between risk and expected returns?”
  + It is a single factor asset pricing model:
    - Describes the risk-return relationship for individual assets
    - An asset’s risk is characterized by its contribution to the risk of an efficient, diversified portfolio
    - The single factor in the market portfolio, and an asset’s relevant risk is its market (systematic) risk
* CAPM Assumptions
  + Investors are rational mean-variance optimizers
    - They form portfolios based on mean returns and variances
  + Homogenous expectations
    - All investors have the same probability distribution of future events
  + All investors can borrow and lend at rf rate
  + Markets are perfectly competitive
  + Investors have a single-period investment horizon
  + No market frictions
    - No taxes
    - No commissions, bid-ask spread, etc.
* CAPM Equilibrium
  + Under CAPM, all investors have the same opportunity set
    - Means they’re using the same method to evaluate the same investments
    - They will invest in the same assets
  + Under these conditions, equilibrium outcome is:
    - All investors hold the same portfolio (M)
    - Market portfolio (M) is the optimal risky portfolio
    - The CAL that runs through M from the risk-free rate is the best CAL
      * This is the CML
* Portfolio M and Expected Returns
  + These results imply that the relevant risk of a particular stock is the risk that it adds to the market portfolio
  + Consider adding a little more of stock i to the market portfolio, borrowing at rate rf to pay for it
    - The change in E(r) is proportional to (ri-rf)
    - The change in standard deviation is proportional to σm
    - This implies that Δreturn/Δrisk is proportional to (ri-rf)/ σm
* Deriving CAPM
* CAPM
  + Describes the expected return on a risky asset as a function of:
    - The risk-free rate
    - The expected market risk premium
    - The systematic risk of the asset
* Example: CAPM
  + Beta for Wells Fargo is 1.1. Expected market risk premium is 7.2% and rf is 1%. What is the expected return for Wells Fargo?
* Security Market Line (SML)
  + Describes the equilibrium relationship between the systematic risk of any individual asset (or portfolio) and its expected return
  + It is the expected return-beta relationship for any asset or portfolio since beta is our measure of systematic risk
* Example: SML
  + Consider the following info for the market portfolio (M) and 2 risky assets, A and B:

|  |  |  |
| --- | --- | --- |
| E(rm) = 8% | σm = 16% | Rf = 3% |
| E(rA) = 12% | σA = 36% | σAM = 460.8 |
| E(rB) = 5% | σB = 10.667% | σBM = 102.4 |

* + Calculate the Betas for A and B
* Mispricing and Abnormal Returns
  + CAPM is an equilibrium model
    - Actual realized returns will often differ from what was predicted
    - Results in mispricing relative to CAPM
    - We can measure mispricing, or abnormal return with alpha:
    - α > 0: positive abnormal return, security is underpriced
    - α < 0: negative abnormal return, security is overpriced
      * Ri = actual return
      * Subtract out CAPM
      * If return is too low, price is too high, and vice versa
      * Want to find positive alphas and avoid negative alphas
* Alpha and the SML
* Example: Mispricing
  + You are evaluating 2 securities to add to your portfolio
    - Stock A has a beta of 0.85 and an average historical return of 11.5%
    - Stock B has a beta of 1.3 and an average historical return of 14%
  + The expected market risk premium is 9.2% and the risk-free rate is 3%. You believe equilibrium rates of return are completely described by CAPM. Based on each security’s alpha, which should you buy?
* CAPM Application #1
  + Investment Performance Evaluation
    - CAPM can provide a benchmark return
      * Based on beta of the portfolio being managed
    - Can compare to the actual, realized return
      * If the manager has exceeded the benchmark, then α > 0
      * If the manager has underperformed the benchmark, then α < 0
    - Keep in mind: CAPM is not the only benchmark available
* CAPM Application #2
  + Capital Budgeting
    - CAPM can provide the required rate of return (hurdle rate) for a capital project, based on the project’s beta
* Estimating CAPM
  + Gives us a function for expected returns:
  + In order to estimate CAPM beta, we want to figure out the correlation between the market risk premium and the expected return
    - The rf will change over time but these changes will have nothing to do with the stock or the market risk
    - We will subtract out rf and focus on the market return
* Estimating CAPM
  + To estimate beta, we use historical returns as our proxy for expected returns
  + We estimate beta by running a regression of excess stock returns on the excess returns of the market portfolio:
  + Note:
    - We are regressing excess returns (risk premium)
    - Ei,t is the error term and is estimated to be 0
* Estimating CAPM
  + Estimating the regression above gives us the security characteristic line (SCL)
    - The slope of this line is estimated by the beta coefficient:
    - The intercept of this line is αi
* SCL Illustrated
* Estimating Beta: Data Notes
  + We are using realized returns as a proxy for expected returns
    - Assuming that you can use past performance to predict future performance
  + Two typical proxies for the market portfolio are:
    - S&P 500
    - A value-weighted index of stocks on the NYSE, Amex, and Nasdaq
  + Both proxies ultimately focus on large US stocks
* Testing CAPM
  + CAPM represents one of the many asset pricing models
    - We can test how well it’s predictions hold up using pricing models
  + Recall the regression equation:
  + In a CAPM world, on average we would expect to see:
    - αi = 0
    - Betai = 1
* Using the BA II Plus for α and Beta
  + Data
    - X = market risk premium for each period
    - Y = security’s excess returns
  + Stat
    - LIN
    - a = α
    - b = beta
* Example: CAPM Regression
  + You have gathered some data on the firm’s returns over the last 3 years. It is presented in the following table:

|  |  |  |  |
| --- | --- | --- | --- |
| Year | RegCo | S&P 500 | T-Bill Rate |
| 1 | 10% | 12% | 1% |
| 2 | 13% | 14% | 1% |
| 3 | 6% | 9% | 2% |

* + What is the firm’s alpha and beta over this time period?
* CAPM Empirical Evidence
  + Empirical tests of CAPM have documented several issues in using CAPM to explain average returns:
    - Large stocks earn returns lower than predicted
    - Small stocks earn returns higher than predicted
    - Growth stocks earn lower returns than predicted
    - Value stocks earn higher returns than predicted
  + CAPM says that systematic risk is the only thing that matters for an asset’s return, so we shouldn’t be able to find systematic trends like this
* Interpreting the evidence
  + Mispricing argument
    - Large stocks are overpriced relative to small stocks
    - Growth stocks are overpriced relative to value stocks
  + Risk premium argument
    - Small stocks have extra risks in relation to large stocks
    - Value stocks have extra risks in relation to growth stocks
      * In both cases, risks are not captured by beta, so investors will demand a risk premium for owning these
* Multi-Factor Models
  + CAPM is a single-factor asset pricing model
    - The factor is the excess return on the market portfolio
    - The empirical evidence suggests this factor alone does not fully explain asset returns
    - There might be other risk factors that influence returns
  + Multi-factor asset pricing models may do a better job, because investors may demand additional risk premiums for additional risk factors
* Fama-French 3 Factor Model
  + Recall the evidence that:
    - Small size stocks outperform large size stocks
    - High book-to-market (value) stocks outperform low book-to-market (growth) stocks
  + Fama and French proposed the following model:
  + Market Factor (RMRF or rm-rf)
    - Market risk premium
  + Size Factor (SMB)
    - The return difference between a portfolio of small stocks and a portfolio of large stocks (small minus big)
    - RSMB = rsmall – rbig
  + Book-to-Market Factor (HML)
    - Return difference between a portfolio of high book-to-market stocks and a portfolio of low book-to-market stocks (high minus low)
    - RHML = rhighBM – rlowBM
* Fama-French 3 Factor Model
  + The factors are the excess returns (or risk premiums) to what are known as factor mimicking portfolios
    - We can’t actually observe the factors, so we built these portfolios to estimate what we think the factor’s value is
    - The mimicking portfolio returns are the same stocks, so we want to set up a model like:
    - The factors loading, or betas, represent each stock’s sensitivity to the factor
    - We may view the additional (SMB and HML) factors as risk premiums similar to the market risk premium
      * Interpret SMB as the risk-premium demanded by investors for taking on the additional risk of holding small firms relative to large firms
      * Interpret HML as the risk-premium demanded by investors for taking on the risk of owning high book-to-market stocks
    - Whether these additional factors measure actual sources of fundamental risk is still a matter of debate
    - Empirically, these factors do help explain average returns
    - There are some potential justifications:
      * Maybe book-to-market is a proxy for financial distress risk
      * High book-to-market stocks (e.g. firms with high book value of equity relative to market value of equity) may potentially be near financial distress
* Using the 3 Factor Model
  + One use for the model would be to tease out the abnormal return (the intercept when we run a regression)
  + Our regression model for investment i would look like:
* Example: FF 3 Factor Model
  + You expect that the risk premia for the 3 FF factors will be:
    - Rm-rf = 6%
    - RSMB = 8%
    - RHML = 3%
  + You regress excess returns for stocks A and B on the 3 factors to estimate each stock’s factor loading (beta) for each of the 3 factors
  + These regressions give the following results

|  |  |  |  |
| --- | --- | --- | --- |
| Stock A: α = 2% | βRMRF = 1.2 | βSMB = -0.5 | βHML = 2 |
| Stock B: α = -0.5% | βRMRF = 0.75 | βSMB = 2 | βHML = 0.5 |

* + If the risk-free rate is 2%, what is the excess return for each stock?
* Arbitrage Pricing Theory
  + Gives a relationship for expected returns that relies on “no arbitrage” requirements
    - It prices securities based on the notion that arbitrage opportunities do not exist in well-functioning capital markets
  + Mispricings can occur
    - Investors are able to profit from these through trading
    - Investor finds two stocks with equivalent risk
    - Investor shorts the lower return and buys the higher return
    - Arbitrage: simultaneous purchase and sale of an asset to profit from the difference in price, results from market inefficiencies
      * Ex. Stock is trading on NYSE for $20 but on London Exchange for $20.05, so you could buy on NYSE and sell on London, making a $0.05 profit
* Arbitrage clarifications
  + A riskless arbitrage opportunity:
    - Occurs when positive payoffs are realized with certainty with zero upfront investment
    - A “zero-investment” portfolio can be formed by shorting the overpriced security and buying the underpriced security
  + The clearest version of a riskless arbitrage is a violation of the law of one price when the same asset sells for different prices in two different markets
* APT Assumptions
  + All securities have finite returns and variances
  + Some people can form well-diversified portfolios
  + No taxes or transaction costs
* CAPM vs. APT

|  |  |
| --- | --- |
| **CAPM** | **APT** |
| Equilibrium model | No arbitrage model |
| Single factor completely describes expected returns | Allows for multiple risk factors to determine expected returns  (doesn’t specify what they are) |
| Many restrictive assumptions  (including the existence of an unobservable market portfolio) | Fewer assumptions  (no market portfolio) |
| Applies to individual assets or portfolios | Applies to well-diversified portfolios |

* Multifactor APT
  + A generalized version of APT with multiple factors describes the expected return of a well-diversified portfolio
  + For example, a model with k factors is given as:
  + Where:
    - βp1…βpk = the factor sensitivities to portfolio p
    - λ1…λk = the risk premiums for each factor
    - λ0 = a constant equal to the risk-free rate, if one exists
  + We could write the model in a more familiar form:
* APT Factors
  + One of the benefits of APT is that we aren’t restricted to any particular factors
  + There are, however, some suggested guidelines for factors:
    - Their impact on asset prices manifests in their unexpected movements
    - They represent undiversifiable influences (typically macroeconomic)
    - Timely and accurate info on these variables is required
    - Their relationship should be theoretically justifiable on economic grounds
* Example ATP Factors
  + Chen, Ross, and Roll (1986) used the following:
    - Surprises in inflation
    - Surprises in GNP
    - Surprises in investor confidence (measured by changes in the default premium of corporate bonds)
    - Surprise shifts in the yield curves
* Notes About APT Factors
  + One of the problems with the factors mentioned in the last slide is that they are “slow”
    - They may be reported monthly or even quarterly, which isn’t necessarily fast enough for today’s market
  + People often use derivative prices in place of these factors in order to have more “active” data
* APT: Factor Mimicking Portfolios
  + We often need to create portfolios whose returns are perfectly positively correlated with the underlying factors, especially if the factor is not a traded asset
    - There are factor mimicking portfolios, which are construed similarly to the 3-factor model
      * Unlike the Fama French mimicking portfolios, we’ll only use a long position bearing the risk
    - The risk premium on the mimicking portfolio serves as a proxy for the factor risk premium:
      * λ1 = E(r1) – rf
      * Where r1 is the return on the mimicking portfolio for factor 1
  + Using this, we can rewrite the general version of the APT using the excess returns of factor mimicking portfolios:
  + We can use as many or whatever factors we want to, with 2 caveats:
    - We must use 1 less factor than the number of assets we have data on
    - We should also avoid the temptation to “p-hack”
      * Stick to variables that “mostly” fit the guidelines
      * Data mining can be dangerous