**Unit 2, Lecture 1-3 Review**

**Risk and Return**

Risk and Return

* Risk: the probability that actual future returns will deviate from expected returns (measure of uncertainty)
	+ Represents the variability of returns
		- Implies a chance for some unfavorable event to occur
* From a finance perspective, risk could be any of the following terms
	+ Sales
	+ COGS
	+ Inventory
	+ Exchange rates

Historical Risk and Return

* Average Return: average percentage return on an investment over a sample time period
* Variance: measures how far returns fall from the mean/average
	+ AKA volatility
	+ A bank CD has almost 0 volatility while a start-up company has a lot of volatility
* A return is the interest or percentage that we earn over a time period (typically a year or month)

$$r\_{t}=return on investment at time t$$

$$r\_{t}=\frac{dollars earned}{dollars invested}$$

$$r\_{t}=\frac{P\_{t}-p\_{t-1}+D\_{t}}{p\_{t-1}}$$

$P\_{t}$ = Price today of investment

$p\_{t-1}$ = Price at beginning of investment

$D\_{t}$ = Dividend paid today

$$r\_{t}=capital gain rate+dividend yield$$

$$capital gain rate=return-dividend yield$$

$$capital gain rate=\frac{P\_{t}-p\_{t-1}}{p\_{t-1}}$$

$$dividend yield=\frac{D\_{t}}{p\_{t-1}}$$

* The average return is then just the total percentage gain or loss divided by the number of returns

$$r\_{avg}=average return$$

$$r\_{avg}=\sum\_{t=1}^{n}\frac{r\_{t}}{n}$$

* The variance, or standard deviation, is how far each return falls from the average
	+ The greater the variance, the greater the volatility or risk of the investment
	+ The SD or variance is also known as the stand-alone risk
		- Stand-alone simply implies the risk associated with only investing in that stock

$$σ^{2}=variance or volatility$$

$$σ^{2}=\sum\_{t=1}^{n}\frac{(r\_{t}-r\_{avg})^{2}}{n-1}$$

Holding Period Return

* HPR = compounded return = total TVM return
* The HPR tells you what you truly made
	+ More realistic than the average return

$$r\_{HP}=holding period return$$

$$1+r\_{HP}=\left(1+r\_{1}\right)\*\left(1+r\_{2}\right)\*…\*(1+r\_{n})$$

Annual Compounded Return

* To find the annual compounded return, we use the HPR formula, but with a geometric return ($r\_{geo}$) instead of annual return ($r\_{t}$)

$$1+r\_{HP}=(1+r\_{geo})^{n}$$

Forward Looking Returns and Risk

* A Random Variable: some measurement that can have a number of possible future outcomes
	+ Ex: temperature, rainfall, company performance
	+ The goal with a random variable is to predict the expected outcome and volatility
* Probability Distribution: a function that assigns probabilities to the various possible outcomes that a random variable can have
	+ There are 2 forms of probability distribution:
		- Discrete: outcomes can take only on a finite number of values
			* Countable
		- Continuous: outcomes can take on infinitely many values over a continuous range of values
			* Ex: stock returns, sales
	+ For either the continuous or discrete probability function, the probabilities must add to 1

Discrete Probability Distribution

* We can use the distributions given in the problem to calculate 2 key statistics:
	+ Expected Value: the measure of central tendency of the distribution (the average value)

$$E\left(r\right)=expected return$$

$$E\left(r\right)=\sum\_{t=1}^{n}(probability of return t)\*(return t)$$

* + Variance: measures the spread of the distribution or the variation in possible outcomes about the expected value

$$σ^{2}=\sum\_{t=1}^{n}(probability of return t)\*(r\_{t}-E\left(r\right))^{2}$$

Continuous Probability Distribution

* Finance uses the continuous, rather than the discrete, probability distribution
* The Normal Distribution is an example of the continuous probability distribution
	+ The Normal Distribution is often used with financial variables such as returns or cash flows
	+ Normal is symmetric around a mean and calculates a z-score that measures how many standard deviations a specific value falls from the mean
		- The safer something is, the more clustered the return (the smaller the spread)
		- The risker something is, the less clustered the return (the larger the spread)
	+ The Normal curve creates confidence intervals for where a random observation will fall
		- Ex: 68% confidence means there’s a 68% chance my future outcome will fall in the predicted range

$$68\%=E(r)\pm σ$$

$$95\%=E(r)\pm 2(σ)$$

Comparing Two Stocks

* To compare two stocks, we have to look at the expected returns
* If the two stocks have the same expected return, then we look at the SD and make a decision
	+ If two stocks have the same expected returns, then choose the stock with the smaller SD because this implies less volatility
* If the two stocks have different expected returns, we can use the coefficient of variation to determine which stock offers the most return per unit of risk
	+ The coefficient of variation tells us how many units of risk you get per unit of return

$$CV=\frac{σ}{E(r)}$$

Lowering Risk with a Portfolio

* The goal of every investor is to build a portfolio and reduce average risk
	+ A portfolio allows you to lower your overall risk
		- “Don’t put all your eggs in one basket”

$$E\left(r\_{p}\right)=expected portfolio return$$

$$E\left(r\_{p}\right)=W\_{A}\*E\left(r\_{A}\right)+W\_{B}\*E\left(r\_{B}\right)+…+W\_{n}\*E\left(r\_{n}\right)$$

$W\_{A }$= Weight in asset A

 = (Dollars invested in asset A) / (Total Dollars Invested)

Formula for Portfolio Risk:

$$σ\_{p}^{2}=\left(W\_{A}^{2}\*σ\_{A}^{2}\right)+\left(W\_{B}^{2}\*σ\_{B}^{2}\right)+2(W\_{A}\*W\_{B}\*δ\_{A,B}\*σ\_{A}\*σ\_{B})$$

$δ\_{A,B}$ = Correlation

* The latter half of the formula represents the $δ\_{A,B} $co-variance, which describes how A and B interact together
* The correlation between two variables is a measure that indicates how much the two variables move or vary together
	+ Correlation falls between -1 and 1 (-1 < $δ\_{A,B} $< 1)
	+ Positive Correlation (0 < $δ\_{A,B}$ < 1)
		- When asset A’s returns are above its average, then Asset B’s return tends to be above its average
			* Assets tend to move in the same direction
	+ Negative Correlation (-1 < $δ\_{A,B}$ < 0)
		- When Asset A’s returns are below its average, then Asset B’s returns tend to be above its average
			* Ideal
	+ Zero Correlation
		- No relationship
	+ \*\*\*The lower the correlation, the greater the diversification, and therefore the lower the risk\*\*\*
		- Most stocks have a correlation between 0.2-0.8

Risk of 3-Asset Portfolio

$$σ\_{p}^{2}=\left(W\_{A}^{2}\*σ\_{A}^{2}\right)+\left(W\_{B}^{2}\*σ\_{B}^{2}\right)+\left(W\_{C}^{2}\*σ\_{C}^{2}\right)+2\left(W\_{A}\*W\_{B}\*δ\_{A,B}\*σ\_{A}\*σ\_{B}\right)+2\left(W\_{A}\*W\_{C}\*δ\_{A,C}\*σ\_{A}\*σ\_{C}\right)+2\left(W\_{B}\*W\_{C}\*δ\_{B,C}\*σ\_{B}\*σ\_{C}\right)$$

* With an increased number of assets, co-variation is all that matters

Market Wide Risk vs. Firm Specific Risk

* What causes volatility?
	+ Market Wide Risk
		- There are economic events that have broad implications and cause all stocks to move up or down together
		- Events like economic recessions or booms, interest rate changes, taxes, political developments, or oil prices will generally impact all firms and thus stock prices
		- These events affect returns in the same direction but not necessarily in the same magnitude (“A high tide moves all boats”)
		- Market risk = non-diversifiable risk = systematic risk
			* An increase in non-diversifiable risk 🡪 an increase in returns
	+ Firm Specific Risk
		- These are economic and business events that impact only one or a few firms at a time
		- Risk that is unique to a specific firm or industry
			* Ex: the release of a new product or technology, a lawsuit, a CEO change
		- Firm specific risk = diversifiable risk = nonsystematic risk

$$σ=market risk+firm specific risk$$

$$σ=systematic risk+nonsystematic risk$$

$$σ=nondiversifiable risk+diversifiable risk$$

Determining the Market Risk Premium

* Market Portfolio: a portfolio that contains all assets in existence
	+ Each asset is held in the same proportion as its value is to the total value in the economy
* Risk of Market Portfolio: the stand-alone risk of the market portfolio is entirely non-diversifiable risk

$$σ\_{m}=risk of market portfolio=systematic risk=nondiversifiable risk of avg. investment$$

* Investors have some expected return to hold the market portfolio
	+ Using this return, we can determine the market risk premium, or the excess return required for the investor to buy the market portfolio

Stocks Relevant Risk

* A stocks relevant risk = a stocks non-diversifiable risk
* A stocks relevant or market risk equals its stand-alone risk times the correlation coefficient that exists between the stocks returns and market portfolio

$$Relevant Risk of stock i=δ\_{i,m}\*σ\_{i}$$

i = stock “i”

$δ\_{i,m}$ = correlation between returns on stock i and returns on the market portfolio

* The higher the $δ\_{i,m}$, the more stock i is influenced by the market portfolio, and the higher the stock i market risk
	+ The higher the $δ\_{i,m}$ the less the diversification

A Stocks Beta

* Beta is a risk index
	+ Beta is a ratio of a stocks relevant risk divided by the relevant risk of the market portfolio

$$β=\frac{relevant risk of stock i}{relevant risk of market portfolio}$$

$$β=\frac{nondiversifiable risk of stock i}{nondiversifiable risk of the average investor}$$

$$β=\frac{δ\_{i,m}\*σ\_{i}}{σ\_{m}}$$

* The higher the $β$, the higher the required return
* The portfolio $β$ is the weighted average of each $β$ in the portfolio

$$β\_{p}=(W\_{A}\*β\_{A})+(W\_{B}\*β\_{B})+…+(W\_{n}\*β\_{n})$$

* Key Point: Beta predicts the expected relationship between the market return and the return on the individual stock
	+ Beta measures how the stock responds to the market
* What do the following Beta’s represent?
	+ $β$ = 1.50
		- When $β$>1, stock i is riskier than the average investment
	+ $β$ = 0.75
		- When $β$<1, stock i is less risky than the average investment
		- If the market is up by 1%, then the average stock i return is up by 0.75%
		- When the market goes up/down, stock i tends to go up/down by a smaller amount
	+ $β$ = 1
		- When $β$=1, stock i has the same risk as the average investment

The CAPM Formula

* The CAPM formula relates a stocks market risk to its required return

$$required return=risk free return+bonus return for nondiversifiable risk$$

$$r\_{i}=r\_{f}+(β\*bonus return for average investment)$$

$$r\_{i}=r\_{f}+β\_{i}[E\left(r\_{m}\right)-r\_{f}]$$

 $r\_{f}$ = risk free return (safe, 0 risk investment)

 $β\_{i}$ = risk index

 $E\left(r\_{m}\right)$ = expected return on market portfolio

 $E\left(r\_{m}\right)-r\_{f}$ = bonus return for average risk investment = market portfolio risk premium

 $β\_{i}[E\left(r\_{m}\right)-r\_{f}]$ = bonus return for risk of stock i

* Beta determines the required return
	+ A safe investment means there is a smaller Beta and a smaller return
* The problem with the CAPM formula is that $β\_{i}$ is a regression coefficient, meaning it uses historical returns
	+ It’s bad that we are focusing on past numbers because company risk never remains the same
	+ In practice, CAPM has some issues but it’s the best we have

The Security Market Line

* The Security Market Line is really the graphical representation of CAPM

$$y=m\*x+b$$

$$r\_{i}=\left[E\left(r\_{m}\right)-r\_{f}\right]\*β+r\_{f}$$

Need 2 Points:

X ($β)$ Y($r\_{i}$)

0 $r\_{i}=r\_{f}$

1 $r\_{i}=E(r\_{m})$

* Buy the stock if actual return > required return
* Sell the stock if actual return < required return
* If actual return > required return…
	+ Investors will buy the stock
		- The stock is undervalued, meaning its priced too low
	+ With people buying the stock, the price will increase, making returns decrease
		- Price will increase until actual return = required return
* If actual return < required return…
	+ Investors will sell the stock
		- The stock is overvalued, meaning its priced too high
	+ With people selling the stock, the price will decrease, making returns increase
		- Price will decrease until actual return = required return

Extra Notes

* Pope bought into Google in 2013 for $345/share and later sold in 2017 at $800/share, but stock prices actually rose even more after that