**Lecture 3-6 Review**

**Time Value of Money**

Opportunities

* When an investment opportunity pops up, there’s specific info you need to know to make a good decision
  + What are the revenues?

What’s the payoff?

* + What are the expenses?
  + Where’s the location?
  + How many investments that are similar fail?
    - What’s the risk of the investment?
  + How long is the investment for?

What’s the opportunity cost?

* + Where else could we invest?

The Idea of TVM:

* A dollar today isn’t the same as a dollar expected tomorrow
* The evaluation of future cash flows is a fundamental theme underlying all areas of finance
* The concept of TVM is quantified through the use of discounted cash flow analysis (DCF)
* DCF techniques allow us to make an exact dollar value adjustment for time and risk so that we can compare the current worth of cash flows arriving at diff times and/or having diff amounts of risk associated with them

DCF Lump Sum Problems

* Lump Sum: a single cash flow at one point in time
  + PV = present value; value of a cash flow today
  + FV = future value; value of a cash flow at some point in the future
  + N = number of periods
    - As N increases, the FV of a cash flow increases because there’s more compounding
  + r = interest rate / return (as a %)
    - i = discount rate = r
      * The risk attached to a cash flow is measured in i
      * Increased risk 🡪 increased return 🡪 increased i = r
        + Risk and return move together
    - As i/r increases, the FV of a cash flow increases because your interest earns more interest
    - Ex of market discount rates:
      * Bank CD = 1%
      * US Bond = 1%-4%
      * Stocks = 6%-9%
      * Venture Capital = 30%-60%
* Single Period Problems
  + Single period problems ask us to move cash flows ahead or back one period
* Multiple Period Compounding Problems
  + If we compound more frequently than 1 year, we are paid more interest more frequently and therefore FV increases
* NPV of a Cash Flow Stream
  + RULE: with TVM, we can compare cash flows if they are valued at the same point in time
    - Ex: Suppose a friend wants to borrow $1000 today. He promises to pay you back $1050 next year. If you want a 10% return on your investment, what should you do?
      * Compare the PV $1000 to the PV of $1050 next year

Simple vs. Compound Interest

* Simple Interest: only earn interest on the original principle
  + Ex: only earn 5% per year on the original $100
* Compound Interest: earn interest on your interest (interest is reinvested)
  + Ex: the $5 you earn in year 1 earns interest going forward

Interest Rates

* Typically, interest rates are expressed as a nominal or annual rate followed by the compound frequency
  + The stated annual rate is called the Annual Percentage Rate (APR)
* If interest is compounded more than once per year, we need to adjust N and i
  + m = number of times that the interest is compounded in a year
    - Using m, we can adjust the time period and the discount rate to match the compounding frequency
    - A higher m increases FV at a decreasing rate
  + r’ = any sub-annual discount rate = (APR) / m
  + n’ = number of sub-annual time periods = (N) x (m)
* Effective Annual Interest Rate (EAR): the yearly compounded rate
  + The EAR finds an annual interest rate (one compound period per year) that provides the identical FV
  + The EAR is the rate if you can only compound once

Cash Flow Streams with DCF

* PV with Uneven Cash Flow Streams
  + Use CF on calculator
* FV with Uneven Cash Flow Streams
  + Find the FV of each cash flow stream and add them up
* Ordinary Annuities
  + Ordinary Annuity: a cash flow stream in which an equal flow (payment) occurs at the end of every period for n periods
  + PV of an Ordinary Annuity
  + FV of an Ordinary Annuity
    - Ex: retirement planning
* Annuity Due
  + Annuity Due: describes a payment stream where the payments are made at the beginning of the period
    - Ex: Leases and retirement withdraws

PV annuity due = (PV ordinary) x (1+r’)

* + With annuities due, each payment is one period sooner than a regular annuity so we earn one extra period of interest (more valuable)
* Amortized Loans
  + Amortized Loan: a loan repaid in equal payments over its life where the payments include interest and principal
    - Ex: mortgages and car loans
  + PMT = Principal Reduction + Interest
    - Interest = (Beg Balance) x (r’)
    - Principal Reduction: how much ownership you have
      * Principle Reduction = PMT - Interest
  + End Balance = Beg Balance – Principle Reduction
  + Principle Paid = Beg Balance – N Balance
  + Summary of Amortization Tables
    - The payment is constant and is found using the standard annuity formula
    - The beg balance for period n is the PV of an annuity with an interest rate of i’ and n’-1 payments
  + When to pay off a loan early:
    - I pay down debt (pay off a loan early) if I cant earn a higher return than the loan rate elsewhere

Comparing Cash Flows

* We can compare cash flows if they are at the same point in time
  + Move the savings into the future (FV)
  + Move withdraws to the present (PV)
* Ex: car value = down payment + loan
  + Down payment = FV of savings
  + Loan = PV of loan payments

Perpetuities and Annuities

* Perpetuity: an annuity that has no end point (N = infinity)
* Growing Annuities: an annuity that increases or decreases by a fixed percentage every year
  + Each payment is g% larger or smaller than the last
* Growing Perpetuity: a series of cash flows with no end point
  + Each cash flow is g% larger than the last
* Deferred Annuity: a constant cash flow stream that starts x years from today
  + 2 part problem:
    - 1. Find the value of the annuity
    - 2. Find the value today